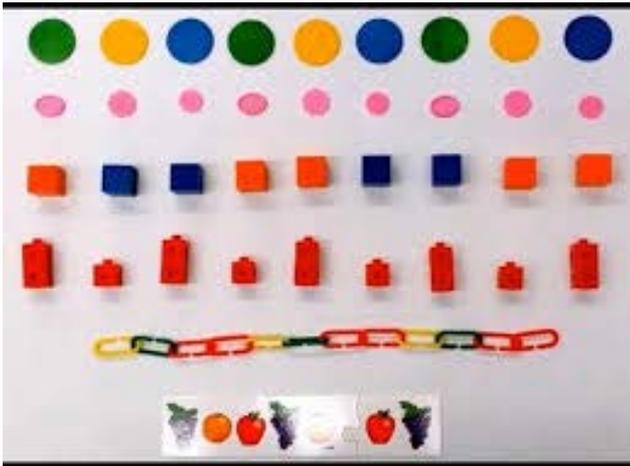


From Patterning to Algebra



Cristina Corbett and Liz Mulholland
November 6, 2017

Image taken from Attribute Patterning youtube



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From Patterning to Algebra

Proposed goals for this session:

- Explore how the curriculum supports growth from patterning to algebra
- Ways we can help students demonstrate pattern growth
- How can we move students from knowing the pattern rule to generating an algebraic expression
- Explore some great problems for patterning that can be used across grade levels



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Patterning The Early Stages



Image taken from creativetots.com

Earliest learners spend a great deal of time making patterns using shape, and colour. Children, through play, are learning that a pattern has a part (core) that repeats. Young children also spend a great deal of time extending patterns of other students, naming patterns and talking about their patterns.



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Grade 2 and 3 Students

Students at the grade 2 and 3 level are making more complex patterns using shape and colour. Students also beginning to explore and extend patterns in numbers using 100s charts, number lines and other tools. They are beginning to state the rule about the pattern.



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In the Junior Grades

Students are working with patterns to...

- describe, represent, extend, and create growing and shrinking patterns using words, tables of values, and graphs;
- represent patterns numerically and geometrically;
- create and extend repeating patterns from reflections, with and without concrete materials;
- understand that patterns are repeated operations, actions or repeated change to an attribute.



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In the Intermediate Grades

In Grade 7 students explore patterns by:

- continue to identify, create, describe, and extend linear growth patterns using a variety of strategies;
- use patterns to relate multiplication of fractions by a whole number to repeated addition of fractions;
- develop formulas for area of trapezoids, volume of right prisms, and surface area of right prisms;
 - use patterns to sort and classify triangles and quadrilaterals, to describe congruent shapes;
 - continue to use patterns to solve simple linear equations;
- describe trends and make inferences from data.



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In the Intermediate Grades

In Grade 8 students explore patterns by:

- continue to identify, create, describe, and extend linear growth patterns;
 - generalize patterns with equations;
 - develop proficiency in multiplying and dividing fractions and integers;
 - develop formulas for circumference and area of circles, volume of right prisms, and surface area of cylinders;
 - sort and classify geometric figures and determine geometric relationships;
- connect patterns to equations;
- continue to describe trends and make inferences from data.



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Visualizing Patterns

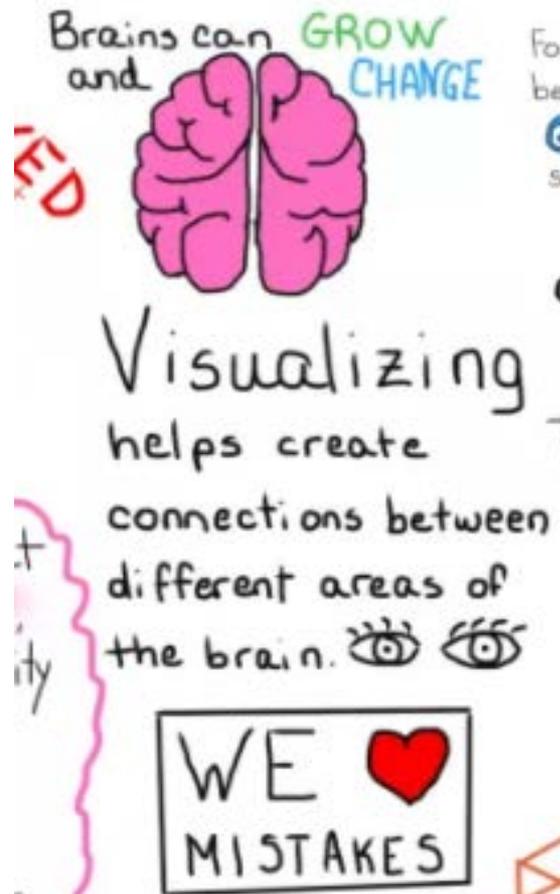


Image from youcubed.org



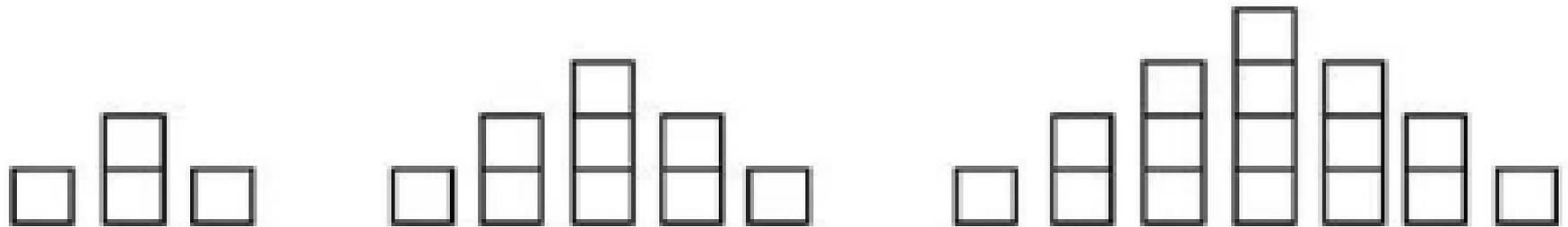
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How do you see the pattern growing?

How do you see the Pattern Growing?



Taken from Jo Boaler week of Inspirational Math @ youcubed.org

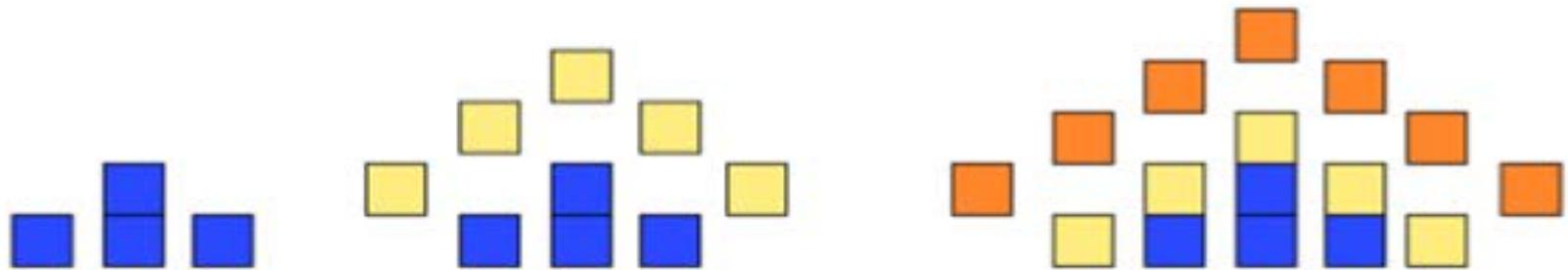


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How can you describe how the pattern grows?



Click on the padlet in the chat box and choose the edit feature to write on representation that shows how you see the growth of the shape:

<https://padlet.com/emulholland/t1zqyl2ny9yl>

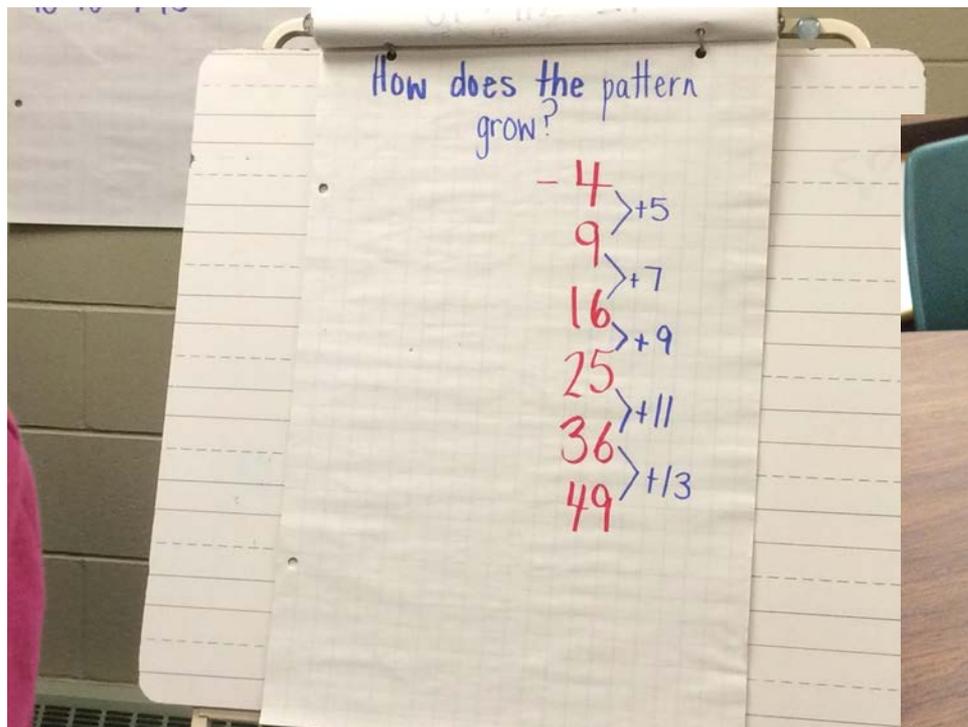


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How A Grade 5 Intervention Group viewed the pattern growth...

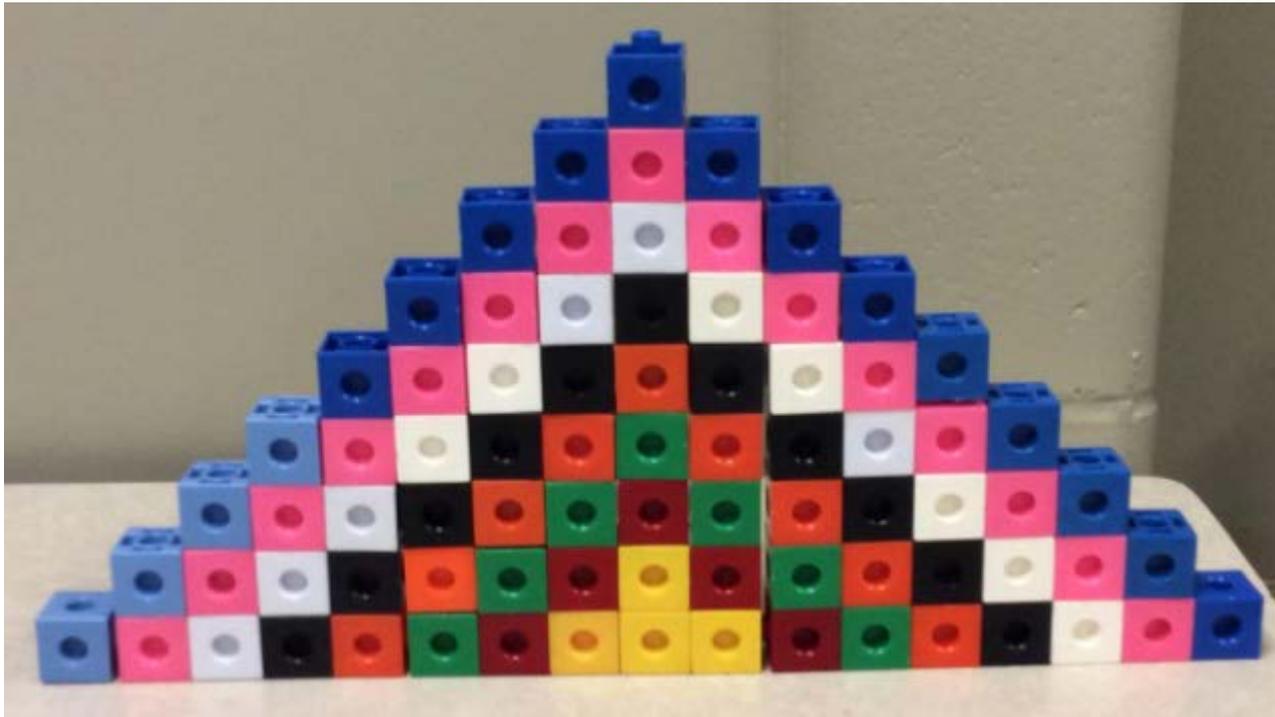


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Representing the Pattern

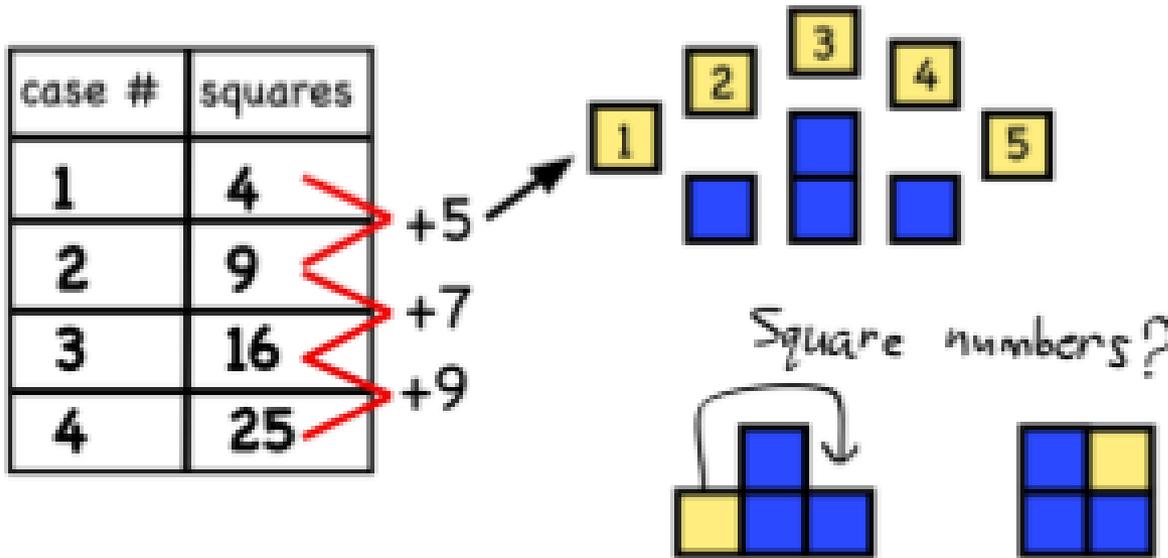


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Exploring the POWER of Visualizing



Images taken from Jo Boaler's book *Mathematical Mindsets*

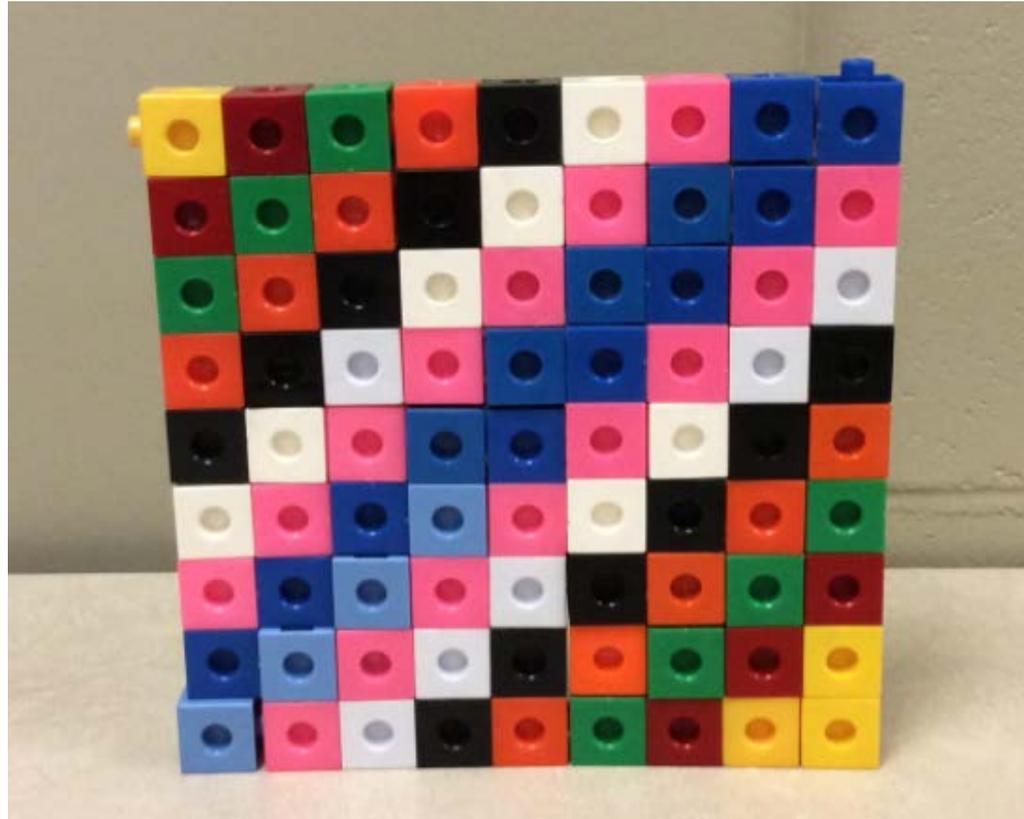


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Reorganizing the Pattern to Make Square Numbers



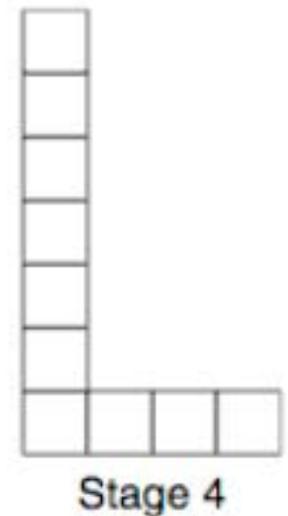
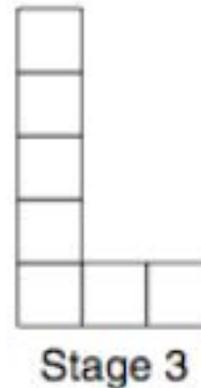
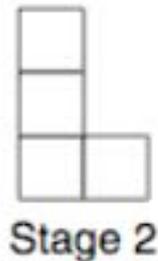
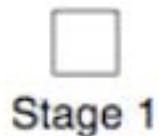
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L-Shaped Problem (Taken from EQAO)

9 Ms. Lewis has 50 blocks. She uses 22 of these blocks to make the pattern shown below.



How many stages will Ms. Lewis be able to complete with the 50 blocks?

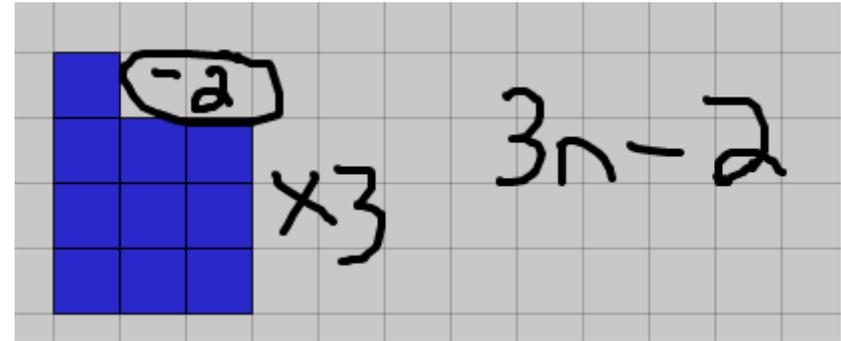
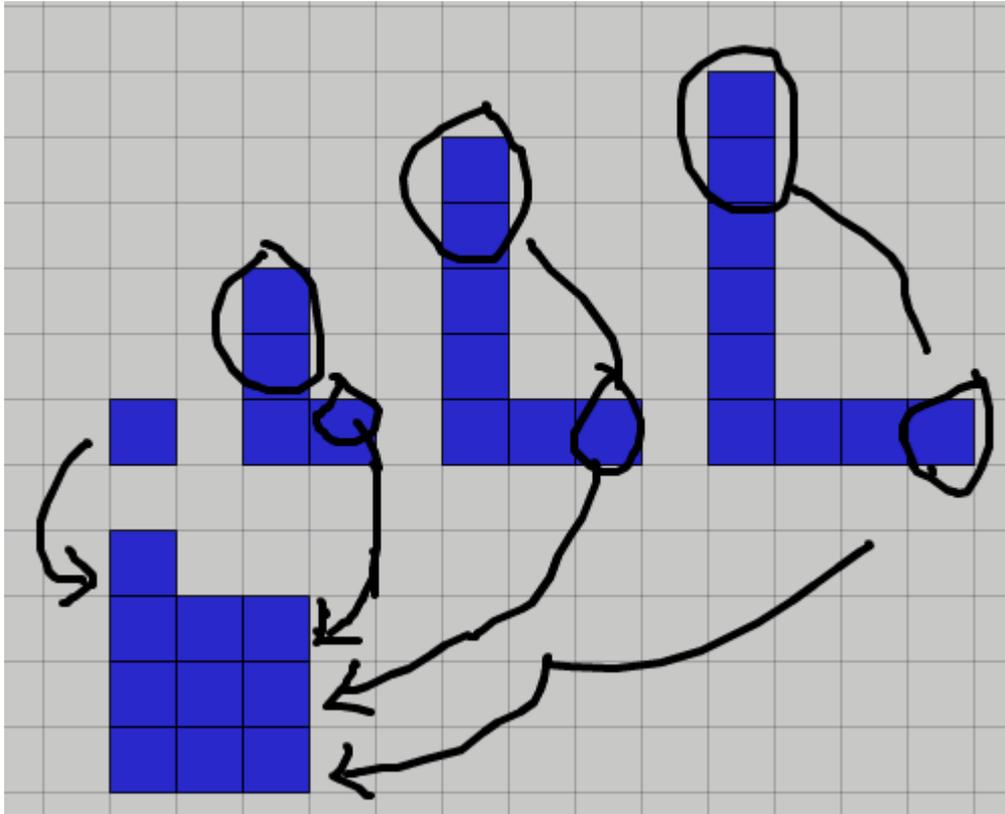


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Reorganizing the Pattern to find the rule



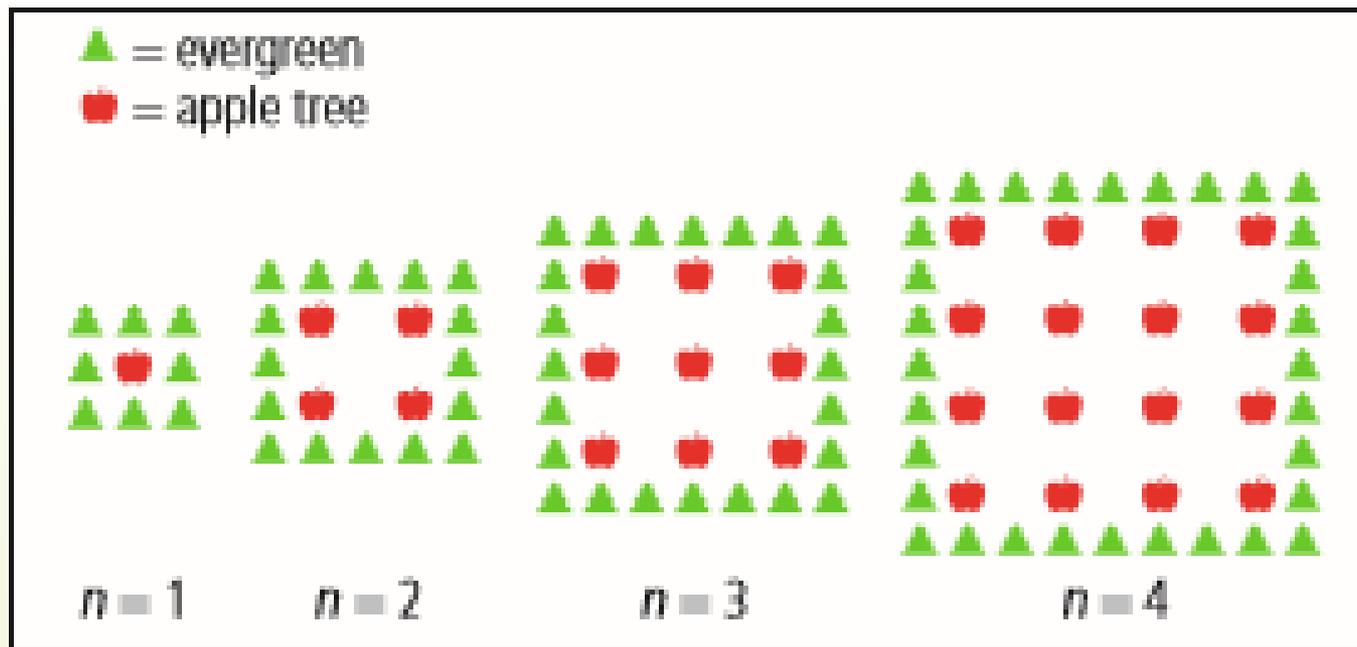
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How can you represent this pattern? How can you show the pattern growth?



Adapted from a PISA prompt

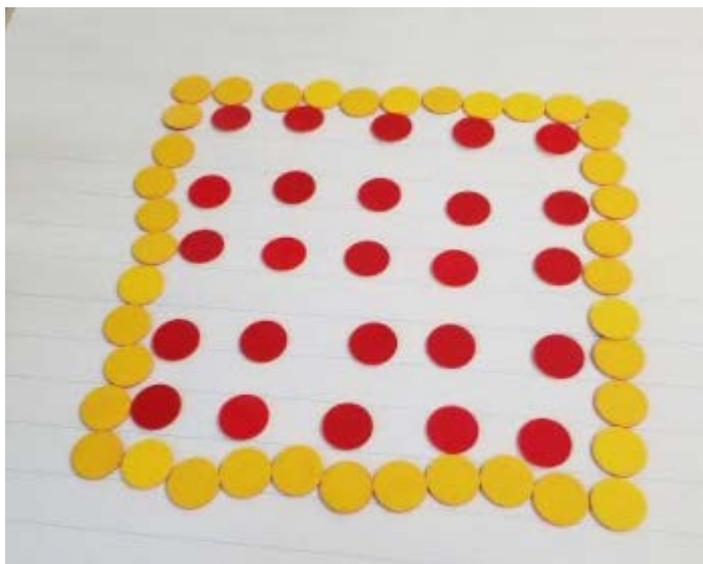


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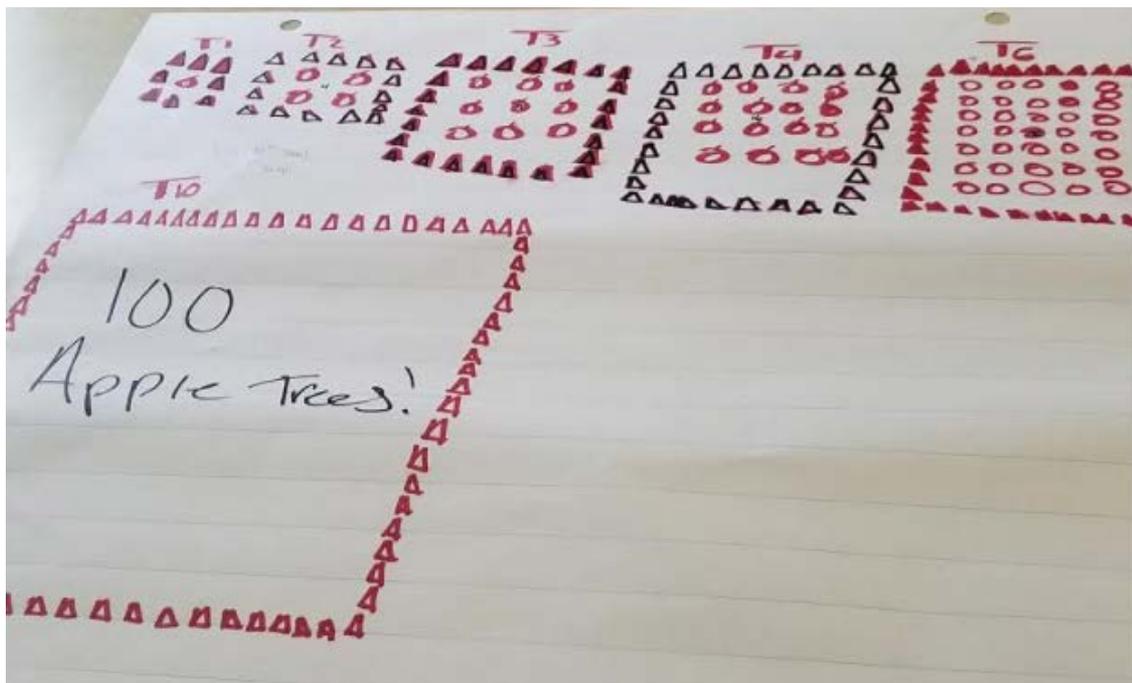
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Student Work from a Grade 8 Class



Creating the next term using hands on materials.



Drawing out how the pattern is growing.



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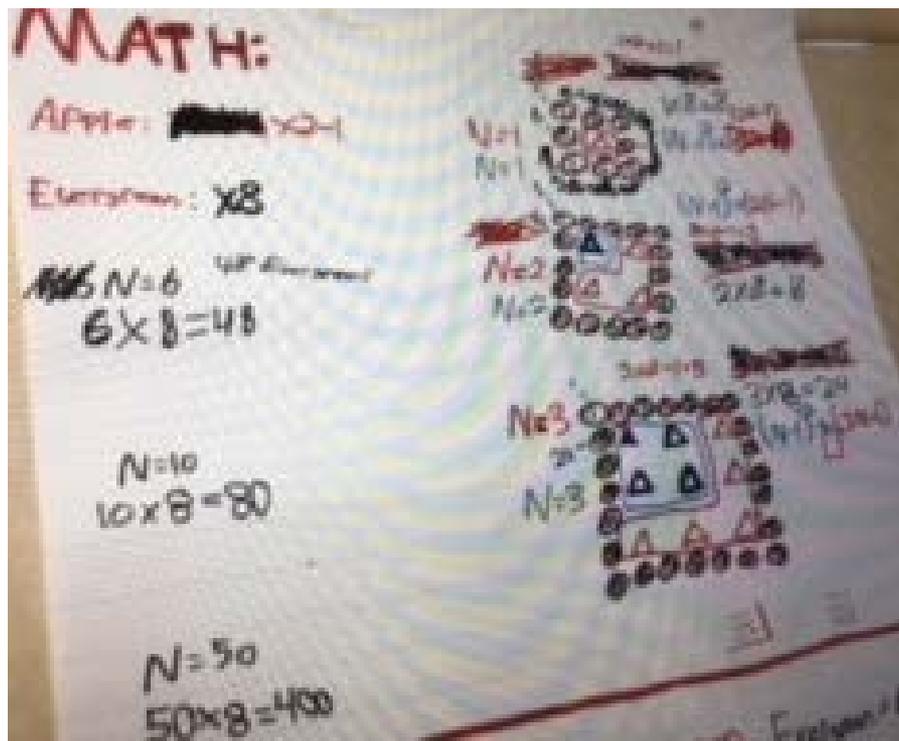
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Student Work from a Grade 8 Class

Students were seeing two parts to the apples - in an L shape and then a smaller square.



What do you see? What do you notice? What do you wonder? ...when you consider the shape?



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Student Work from a Grade 8 Class

Apple Tree	ever green
1 = 1	1 = 8
2 = 4	2 = 16
3 = 9	3 = 24
4 = 16	4 = 32
5 = 25	5 = 40
6 = 36	6 = 48
10 = 100	10 = 80
50 = 100 2500	50 = 400

Demonstrating pattern growth using a table or T-Chart

$\Delta = +8$ $\delta = x^2$
 The Δ goes up by 8
 n8 The δ multiplying the term
 example: 3×3

Finding the algebraic equation.
Does it work every time?
For every term?



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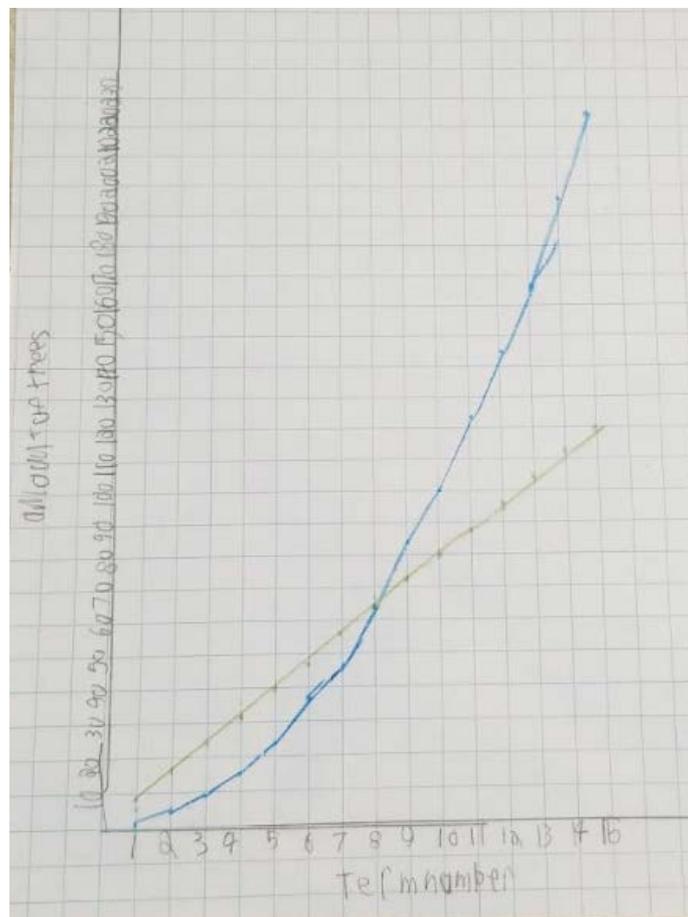
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Student Work from a Grade 8 Class

Students graphed their results to show the relationship between the growth of the patterns.



What do you see? What do you notice? What do you wonder? ...when you represent the patterns as a line graph?



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Actions to Develop Algebraic Reasoning

Students must be given multiple opportunities to develop algebraic reasoning by engaging in the following mathematical processes:

- Offering and testing conjectures
- Justifying and proving
- predicting

Taken from Paying Attention to Algebraic Reasoning p. 13



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Algebraic Reasoning Across Strands and Grades

Algebraic reasoning can be fostered through activities that encourage students to go beyond numeric reasoning to more general reasoning about relationships and quantities.

Taken from Paying Attention to Algebraic Reasoning p. 19



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Algebraic Reasoning Across Strands and Grades

Here is an example of a question related to measurement that could be used with students from K to 12.

You want to build a rectangular garden. What are some possible gardens that have a perimeter of 24 metres?

Taken from Paying Attention to Algebraic Reasoning p. 19



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How can students represent patterns?

Exploring Patterns

We can draw out the patterns.

We can build patterns with tiles.

We can find the pattern rule.

We can use a table of values.

We can create a graph to show pattern growth.

We can generalize to create an equation.

Figures	Squares
1	1
2	3
3	6
4	10
5	15
6	21

$55 \times 55 = 3025$
 $3025 \div 2 = 1512.5$
 $1512.5 + 1512.5 = 3025$
 55 term = 1512.5 tiles

$$n^2 - \frac{(n-1)n}{2}$$

n = term # or figure #

IF YOU
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SOU
AL
IF YOU F
LEAVE

FOR
South
Place / Bla

YOUR ALTERNATE CIT

SMOKE ALA

Figures	Inc
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	10
20	20
*	*



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200+ Patterns

visualpatterns.org

<http://oame.on.ca/mathies/learningTools.php>



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Resources

Paying Attention to Algebraic Reasoning K-12
Monograph

Open Questions for the Three Part Lesson:
Patterning and Algebra, Marian Small

The Guide to Effective Instruction: Patterning and
Algebra Grades 4-6



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Resources Continued

Mathies - patterning tools

<http://oame.on.ca/mathies/learningtools.php>

http://www.edugains.ca/resourcesMath/VideoLibrary/AdobePresenter/AP_AlgebraicReasoning/

Mathematical Mindsets, Jo Boaler

From Patterning to Algebra, Ruth Beatty



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For more information contact us...

Follow us on



@ccorbett819

@mulhollandl

Or email at

emulholland@necdsb.ca

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We love to chat about math and share our learning!



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